LESSON 5. 4b

Solving Equations With Rational Exponents

Today you will:

- Solve equations that have rational exponents.
- Practice using English to describe math processes and equations

Let’s summarize our process for solving equations with radicals:

1. Simplify the expression
2. Isolate the radical
3. Eliminate the radical … raise both sides to same power as root

Rules:

- Preserve the balance
    - If you do it to one side do it to the other
- Simplifying
    1. Simplify/reduce inside the radical
    2. No negative exponents
    3. No fractions in radicals
    4. No radicals in denominator
- $\pm$
    - Only consider when solving an equation
    - …and start with even root
    - Don’t need if squaring both sides (raise to even power)

Solve $\sqrt{x+2} + 1 = \sqrt{3-x}$.

SOLUTION

Gotta do it to the **ENTIRE** other side too!

If you do it to this side…

$\sqrt{x+2} + 1 = \sqrt{3-x}$	Write the original equation.
$(\sqrt{x+2} + 1)^2 = (\sqrt{3-x})^2$	Square each side.
$x + 2 + 2\sqrt{x+2} + 1 = 3 - x$	Expand each side and simplify.
$2\sqrt{x+2} = -2x$	Isolate radical expression.
$\sqrt{x+2} = -x$	Divide each side by 2.
$(\sqrt{x+2})^2 = (-x)^2$	Square each side…again ☺
$x + 2 = x^2$	Simplify.
$0 = x^2 - x - 2$	Write in standard form.
$0 = (x-2)(x+1)$	Factor.
$x - 2 = 0$ or $x + 1 = 0$	Zero-Product Property
$x = 2$ or $x = -1$	Solve for x.

Bah!
Got another radical …
what do we do?

1) Simplify,
2) Isolate the radical
3) Get rid of it

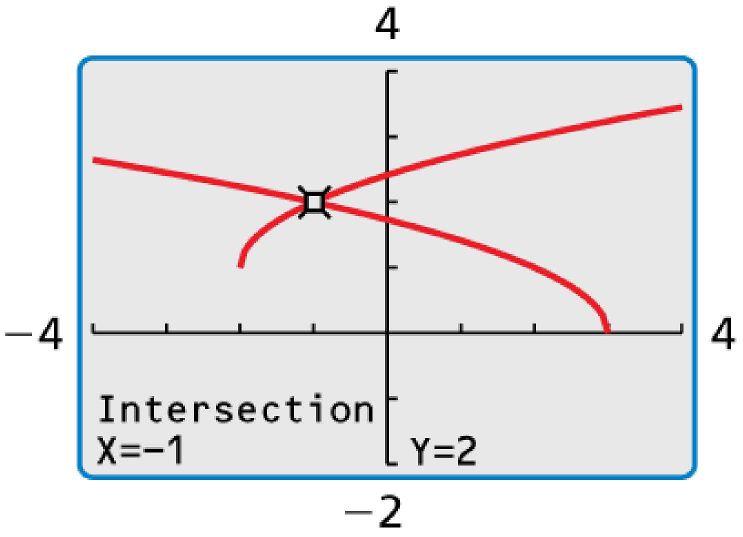
…just like we did before

Solve $\sqrt{x+2}+1=\sqrt{3-x}$.

$x = 2$ or $x = -1$ Solve for x.

ANOTHER METHOD

You can also graph each side of the equation and find the x-value where the graphs intersect.



Check $\sqrt{2+2}+1 \stackrel{?}{=} \sqrt{3-2}$

$\sqrt{4}+1 \stackrel{?}{=} \sqrt{1}$

$3 \neq 1$ ✗

$\sqrt{-1+2}+1 \stackrel{?}{=} \sqrt{3-(-1)}$

$\sqrt{1}+1 \stackrel{?}{=} \sqrt{4}$

$2 = 2$ ✓

The apparent solution $x = 2$ is extraneous.
So, the only solution is $x = -1$.

Rewrite $\sqrt[3]{2x} = 4$ using exponents. $\Rightarrow$ $\mathbf{(2x)^{1/3} = 4}$

Same equations … different form

Which means the process used to solve
one can be used to solve the other.

1. Simplify the expression
2. Isolate the exponent
3. Eliminate the exponent … wait, how do we do that?

What does it mean to "eliminate the exponent?"

Do we literally mean get rid of it? Make it go away?

Hint: what is the exponent on the variable in this equation? $2x + 1 = 0$

…is there actually an exponent hiding there? Duh, yeah … $2x^1 + 1 = 0$

"***Eliminate the exponent"*** means turn it into 1.

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How do you “eliminate” a rational exponent? Turn it into 1.

Turn $\frac{2}{3}$ into 1.

Multiply the fraction by its reciprocal.

$$\frac{2}{3} \cdot \frac{3}{2} = 1$$

Solve $(2x)^{1/3} = 4$.

SOLUTION

$$(2x)^{1/3} = 4$$

Write the original equation.

$$\left((2x)^{1/3}\right)^{3/1} = (4)^{3/1}$$

Raise each side to the third power.

$$2x = 64$$

Divide each side by 2.

$$x = 32$$

The solution is $x = 32$.

Check

$$(2 \cdot 32)^{1/3} \stackrel{?}{=} 4$$

$$64^{1/3} \stackrel{?}{=} 4$$

$$4 = 4 \checkmark$$

Solve $(2x)^{3/4} + 2 = 10$.

SOLUTION

$(2x)^{3/4} + 2 = 10$	Write the original equation.
$(2x)^{3/4} = 8$	Subtract 2 from each side.
$[(2x)^{3/4}]^{4/3} = 8^{4/3}$	Raise each side to the four-thirds.
$2x = 16$	Simplify.
$x = 8$	Divide each side by 2.

The solution is $x = 8$.

Check

$$(2 \cdot 8)^{3/4} + 2 \stackrel{?}{=} 10$$

$$16^{3/4} + 2 \stackrel{?}{=} 10$$

$$10 = 10 \checkmark$$

Solve $(x + 30)^{1/2} = x$.

SOLUTION

$(x + 30)^{1/2} = x$	Write the original equation.
$[(x + 30)^{1/2}]^2 = x^2$	Square each side.
$x + 30 = x^2$	Simplify.
$0 = x^2 - x - 30$	Write in standard form.
$0 = (x - 6)(x + 5)$	Factor.
$x - 6 = 0$ or $x + 5 = 0$	Zero-Product Property
$x = 6$ or $x = -5$	Solve for x.

Check $(6 + 30)^{1/2} \stackrel{?}{=} 6$

$36^{1/2} \stackrel{?}{=} 6$

$6 = 6$ ✓

$(-5 + 30)^{1/2} \stackrel{?}{=} -5$

$25^{1/2} \stackrel{?}{=} -5$

$5 \neq -5$ ✗

The apparent solution $x = -5$ is extraneous.
So, the only solution is $x = 6$.

Homework

Pg 266, #21-36